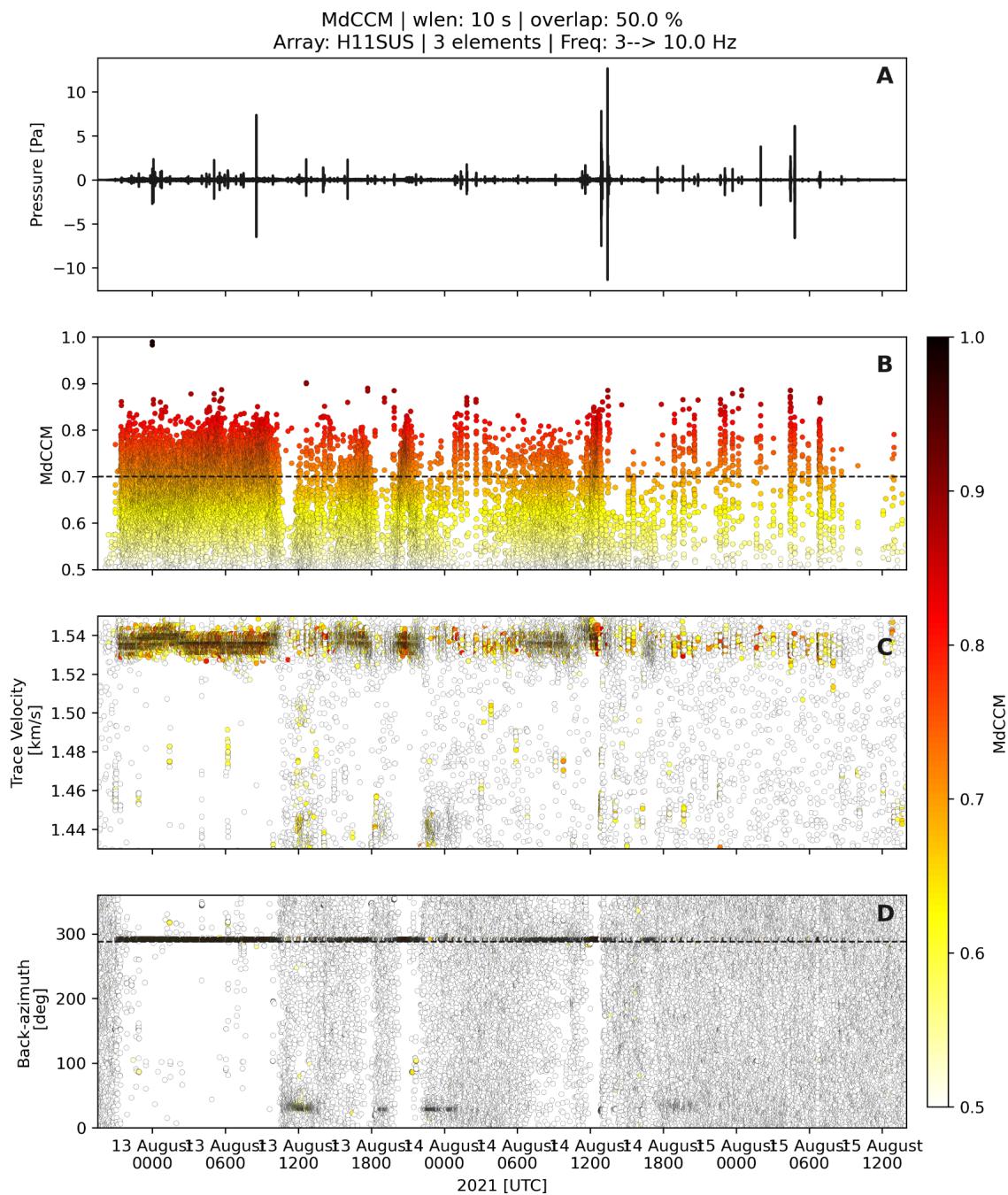


This Supplementary Material accompanies the article

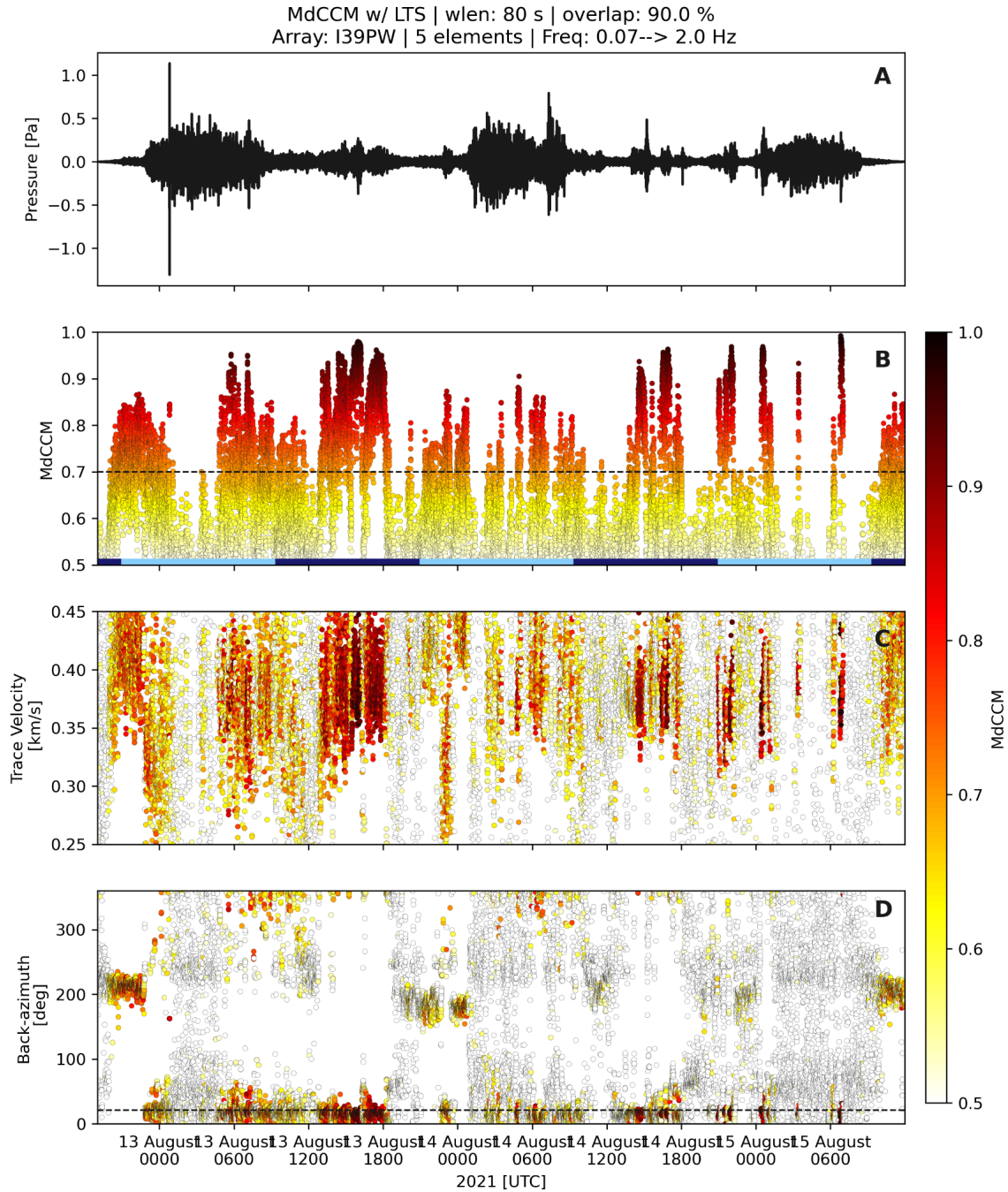
McKee, K., Fauria, K. E., Caplan-Auerbach, J., Assink, J. D., Louis, M., Mittal, T., Maeno, F., Ichihara, M., Metz, D. and Vagasky, C. (2026) "Eruptive flux drives atmospheric lightning rates and submarine hydroacoustic signals during the 2021 Fukutoku-Oka-no-Ba eruption", *Volcanica*, 9(1), pp. 269–288. doi: 10.30909/vol/ebif5496.

Supporting Information:

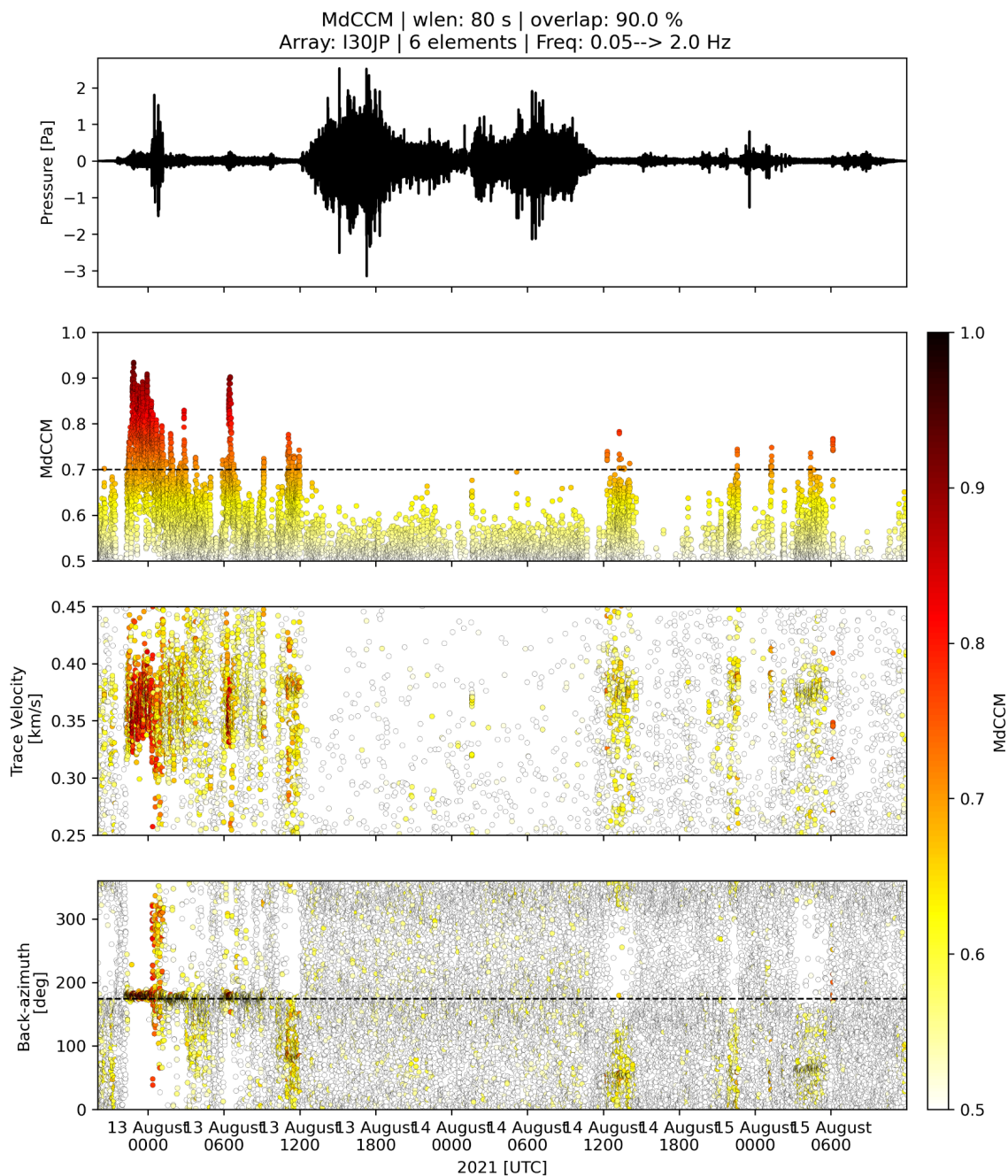
1. **Supporting Figure S1.** H11S Array processing results
2. **Supporting Figure S2.** IS30 Array processing results
3. **Supporting Figure S3.** IS39 Array processing results
4. **Supporting Figure S4.** Lightning and Hydroacoustic Detection Rate Cross-Correlation
5. **Supporting Figure S5.** Lightning Detection Rate and Volume Flux Cross-Correlation
6. **Supporting Figure S6.** Hydroacoustic Detection Rate and Volume Flux Cross-Correlation
7. **Supporting Figure S7.** H11S Array processing results on 2021 08 12 at 21:00 UTC
8. **Supporting Figure S8.** H11S Array processing results on 2021 08 14 at 12:20 UTC
9. **Supporting Figure S9.** Multiparameter data of the FOB Eruption and an Alaskan earthquake, magnitude 6.6.
10. **Supporting Figure S10.** Infrasound propagation modeling
11. **Supporting Figure S11.** Hydroacoustic propagation modeling at 50 Hz
12. **Supporting Figure S12.** Hydroacoustic propagation modeling at 10 Hz
13. **Supporting Figure S13.** Hydroacoustic propagation modeling at 5 Hz
14. Discussion on Density-driven vs. Passive transport
15. **Supporting Figures S14-S21.** H11S array processing results with different parameters



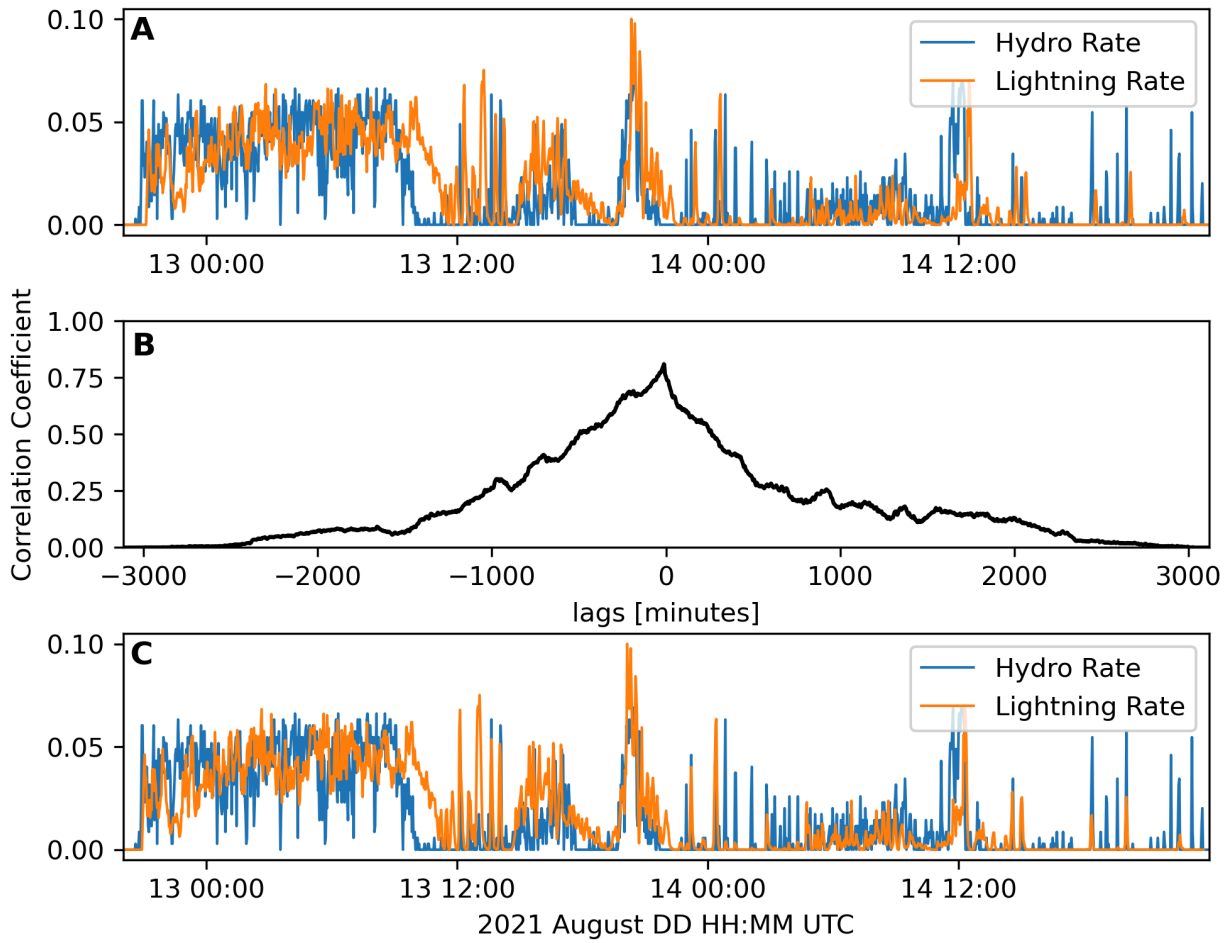
Supporting Figure S1. Array processing result for hydroacoustic array H11S. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags.



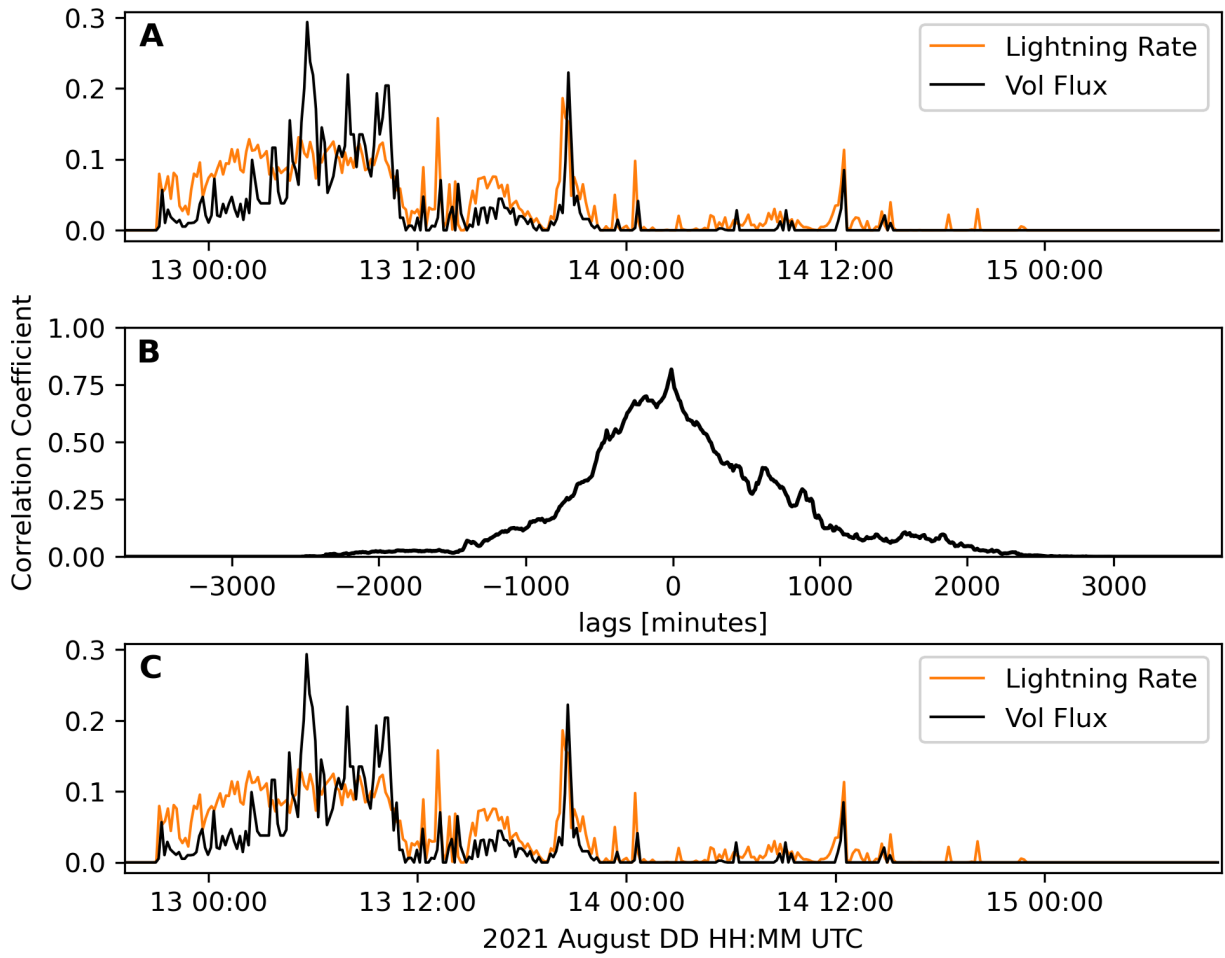
Supporting Figure S2. Array processing result for infrasound array I39. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags.



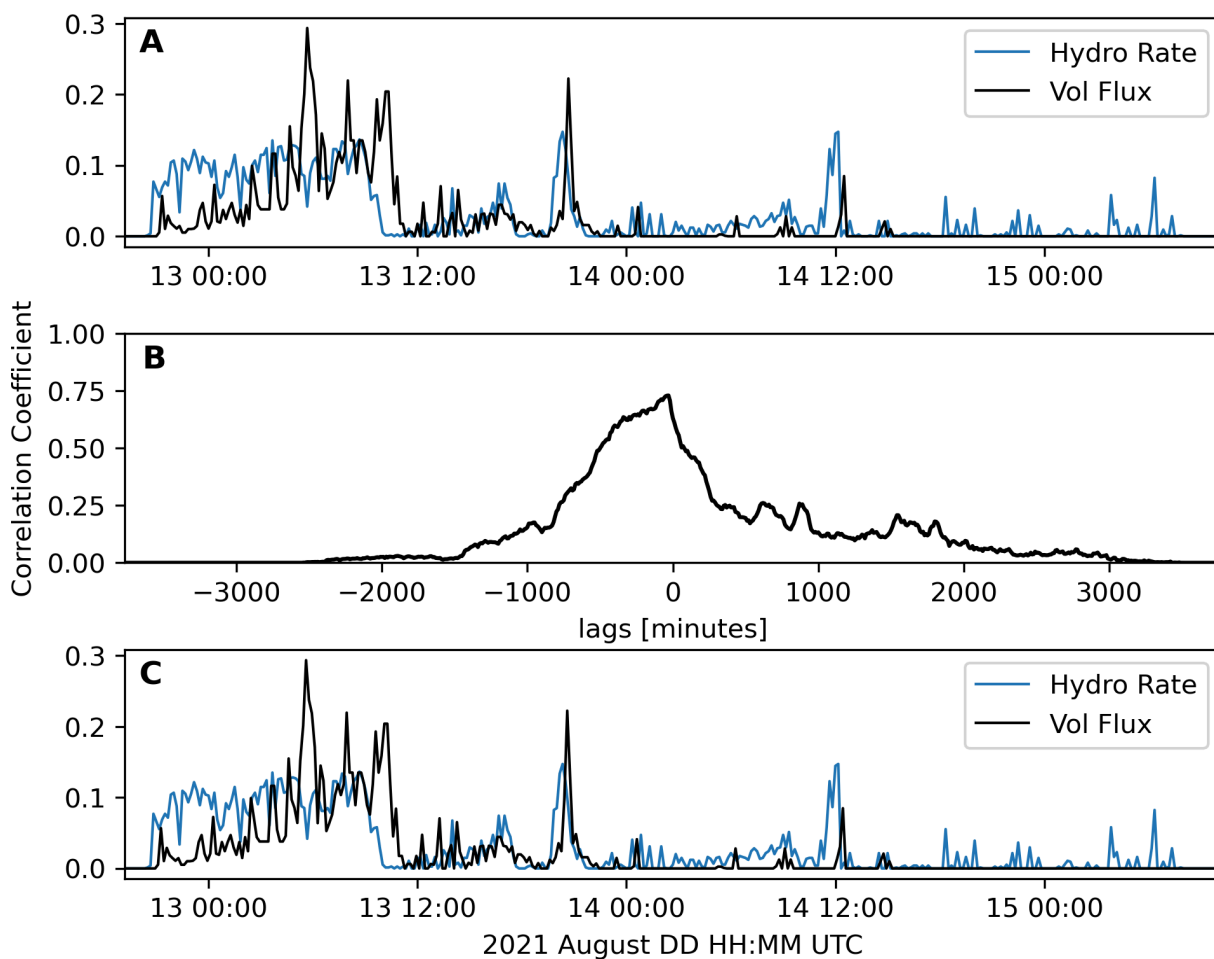
Supporting Figure S3. Array processing result for infrasound array I30. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags.



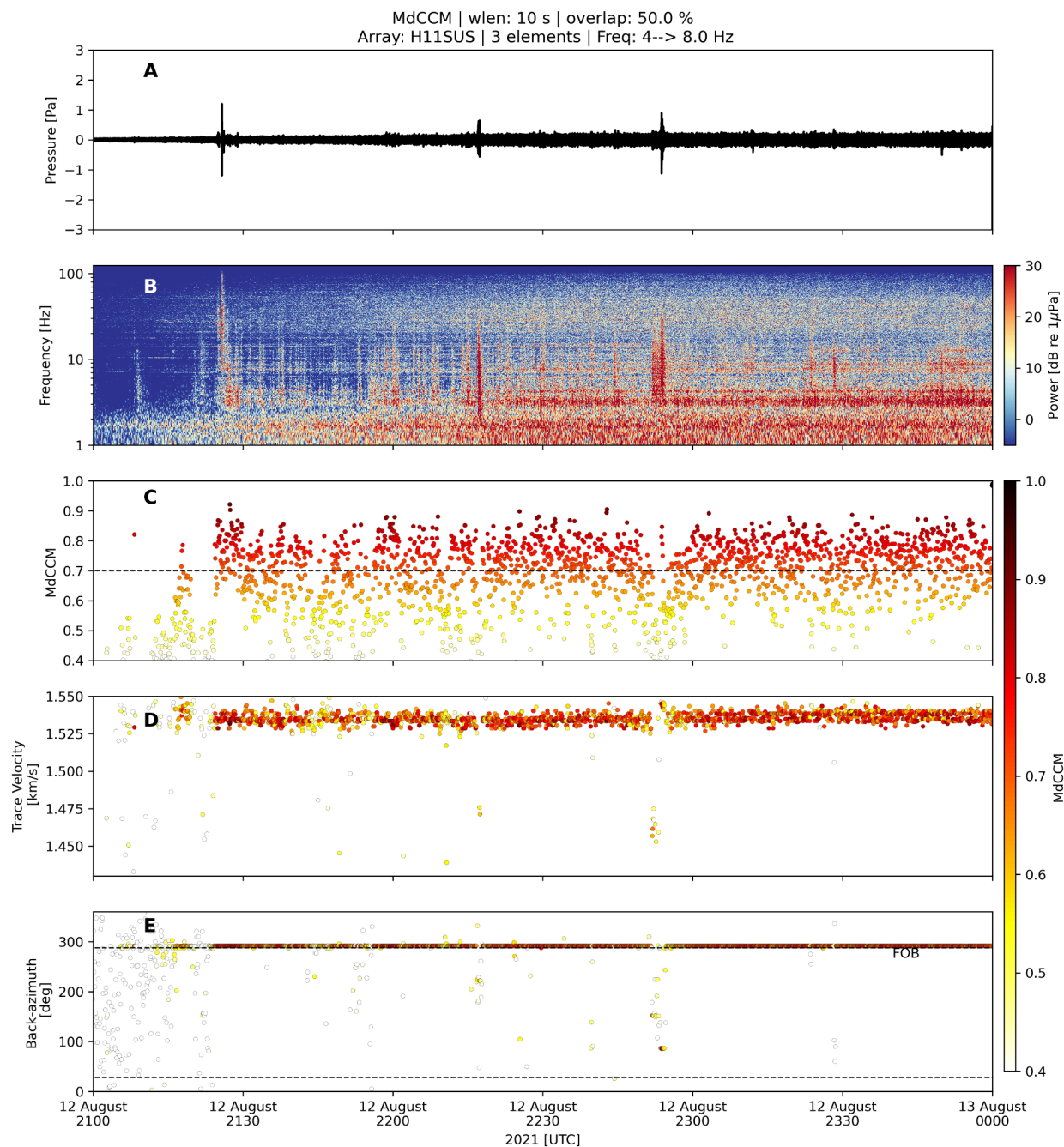
Supporting Figure S4. Hydroacoustic and lightning detection rate cross-correlation result. A) Hydroacoustic and lightning normalized detection rates. B) Pearson correlation coefficient. C) Hydroacoustic and lightning-normalized detection rates with lightning data shifted earlier in time by the 720-second lag.



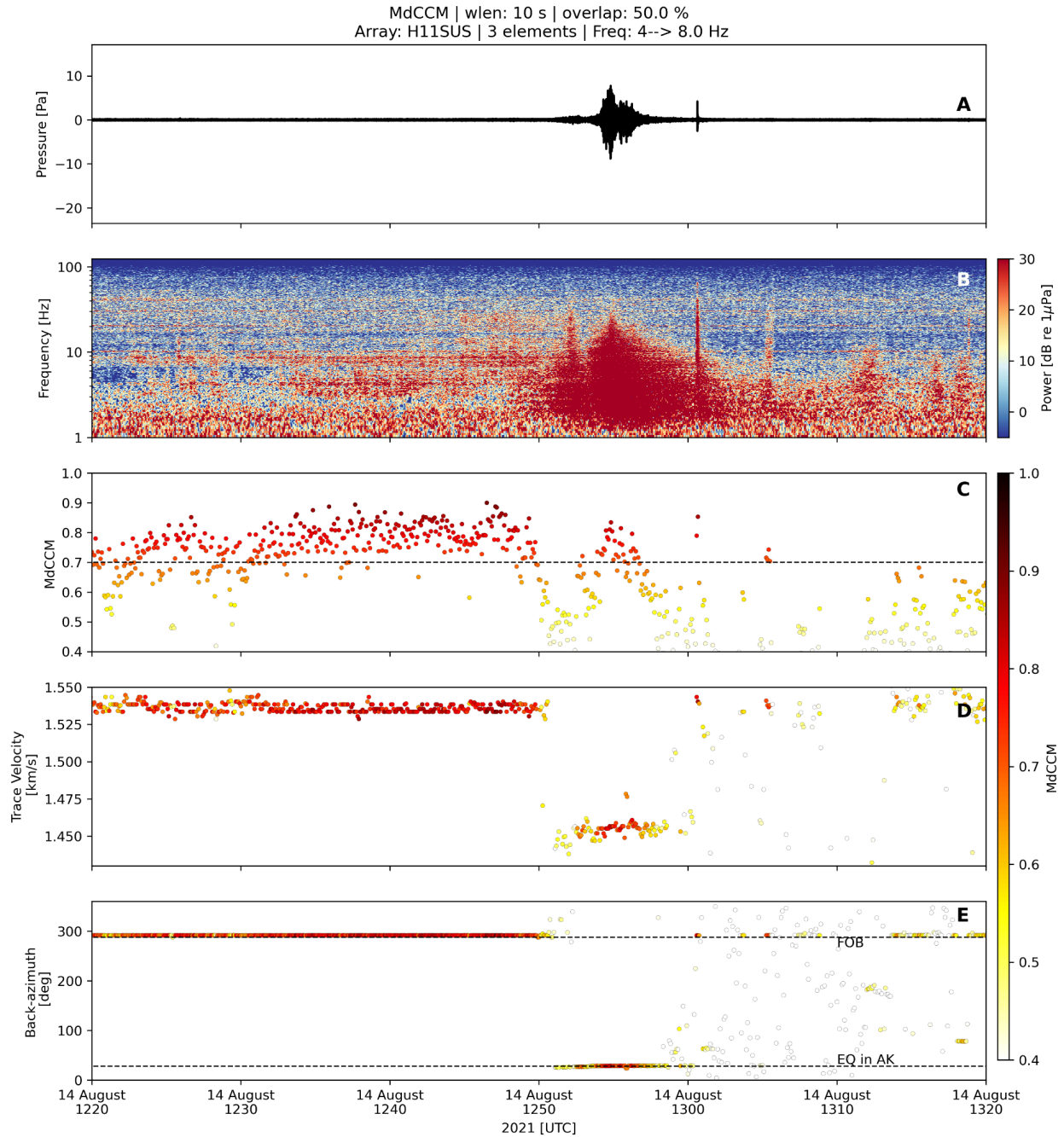
Supporting Figure S5. Lightning detection rate and volume-flux cross-correlation results. A) Normalized lightning detection rate and volume flux. B) Pearson correlation coefficient. C) Normalized lightning detection rate with volume flux data shifted earlier in time by the 600-second lag.



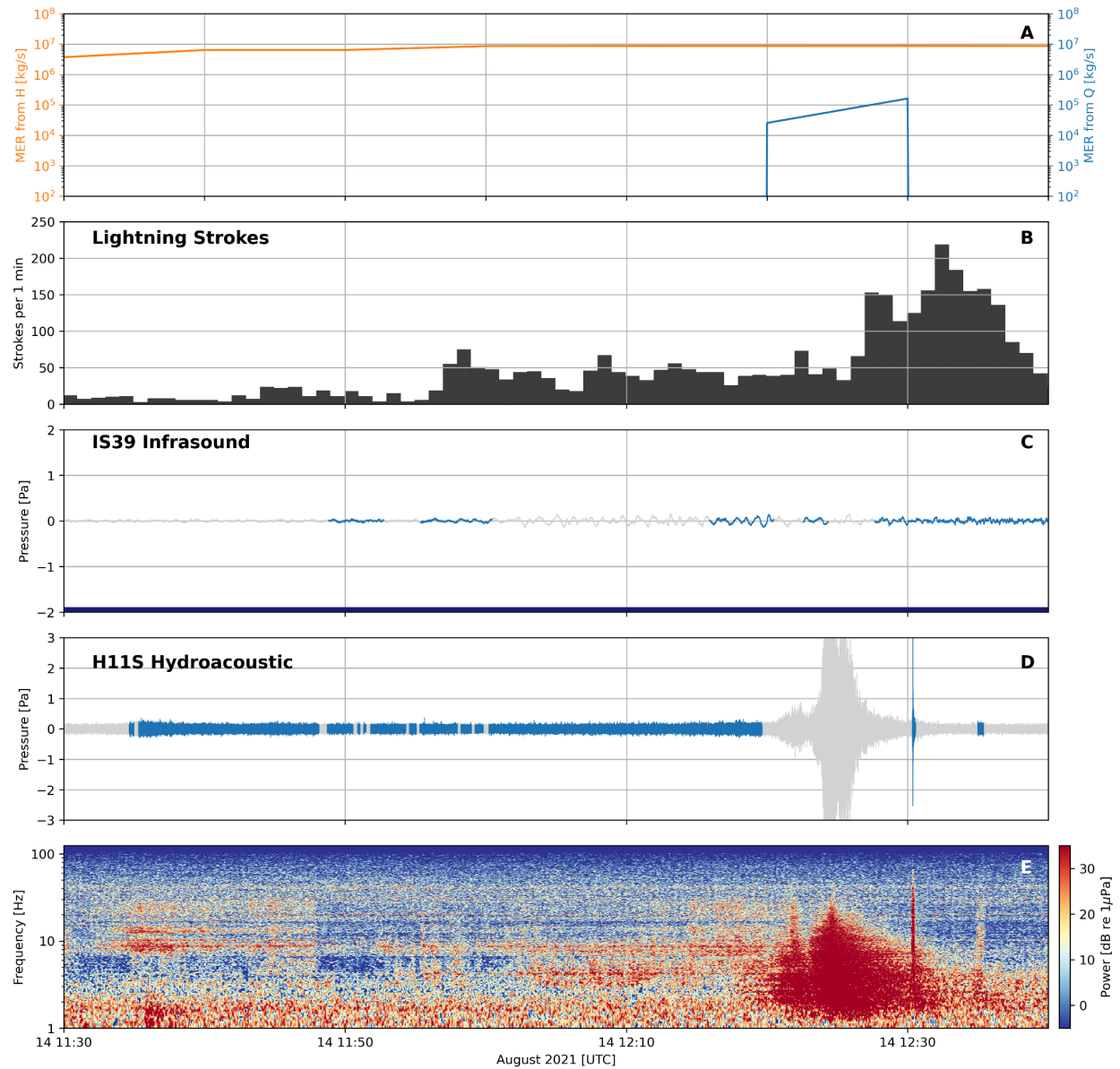
Supporting Figure S6. Hydroacoustic detection rate and volume-flux cross-correlation results. A) Normalized hydroacoustic detection rate and volume flux. B) Pearson correlation coefficient. C) Normalized hydroacoustic detection rate with volume flux data shifted earlier in time by the 1200-second lag.



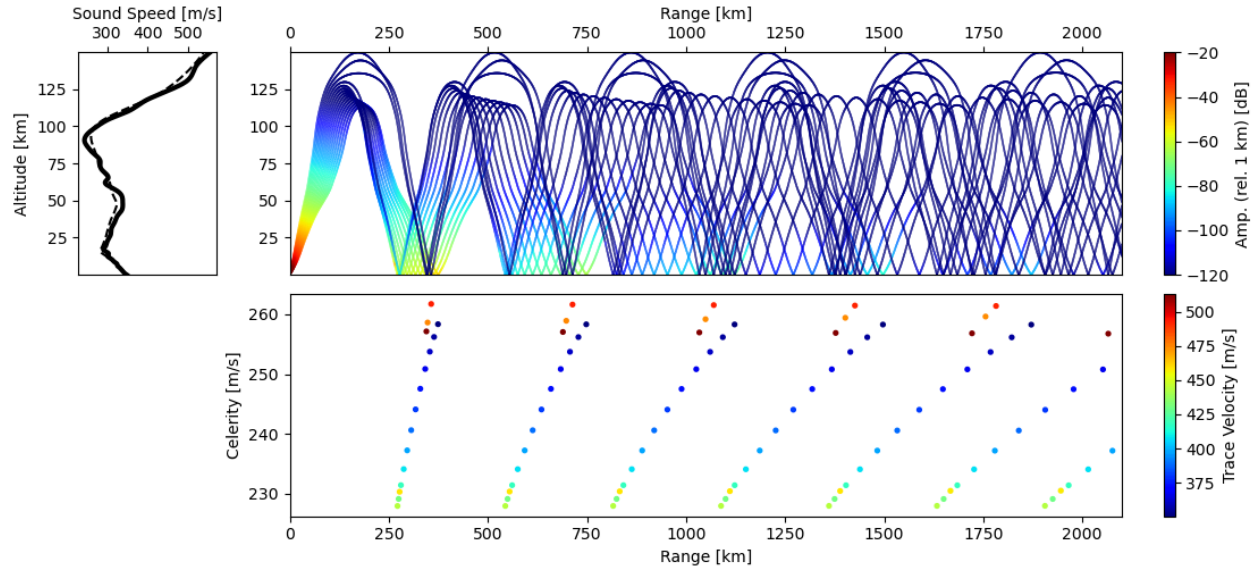
Supporting Figure S7. H11S Array processing result for 12 August 2021 at 2100 to 13 August 2021 at 0000 UTC. Note that these times have not been adjusted for travel time. A) Beamform from 3-element array data for FOB back-azimuth. B) Spectrogram of Hydroacoustic data. C) Median cross-correlation maximum for each window. D) Trace velocity across the array. E) Back-azimuth resolved from cross-correlation lags.



Supporting Figure S8. H11S Array processing result for 14 August 2021 from 1220 to 1320 UTC showing detentions from the FOB eruption, and then after 1250 UTC detections from an earthquake in Alaska. Note that these times have not been adjusted for travel time. A) Beamform from 3-element array data for FOB back-azimuth. B) Spectrogram of Hydroacoustic data. C) Median cross-correlation maximum for each window. D) Trace velocity across the array. E) Back-azimuth resolved from cross-correlation lags.

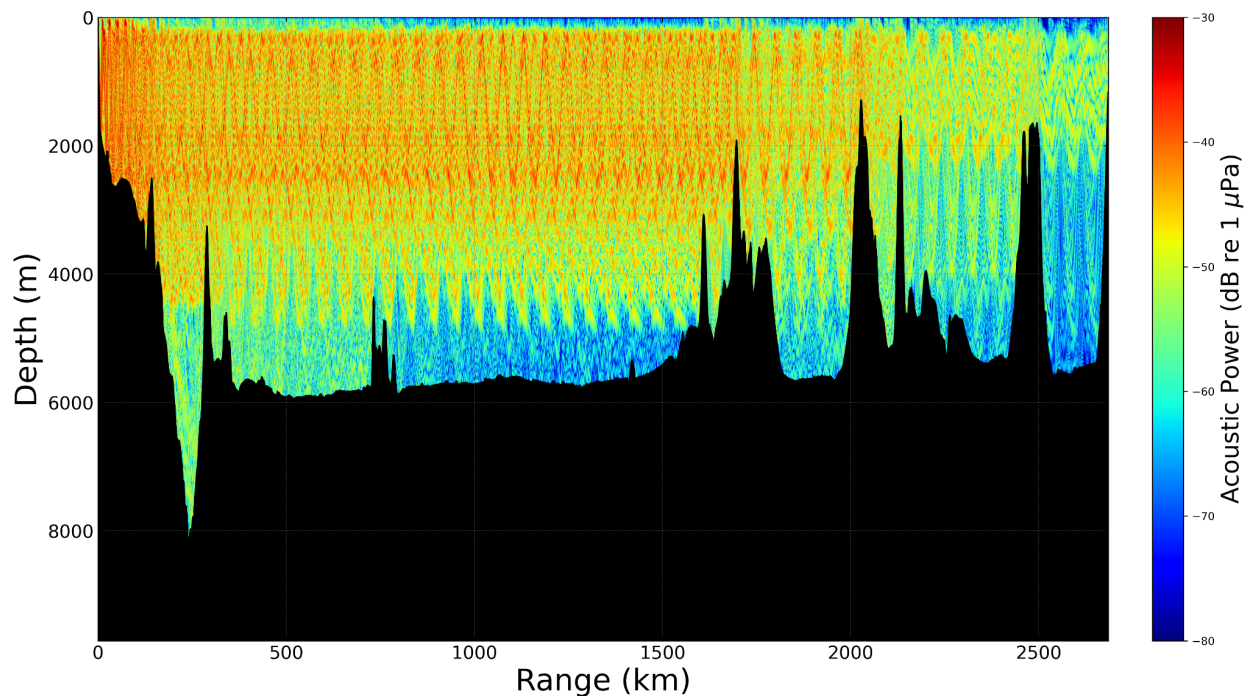


Supporting Figure S9. The hours leading up to the arrival of a magnitude 6.6 Alaskan earthquake. A) Shows the MER from plume height and volume flux. B) Shows lightning rate per minute. C) Show IS39 infrasound beamform with sound from FOB plotted in blue. D) Hydroacoustic beamform with sound from FOB in blue. E) Spectrogram of Hydroacoustic data.

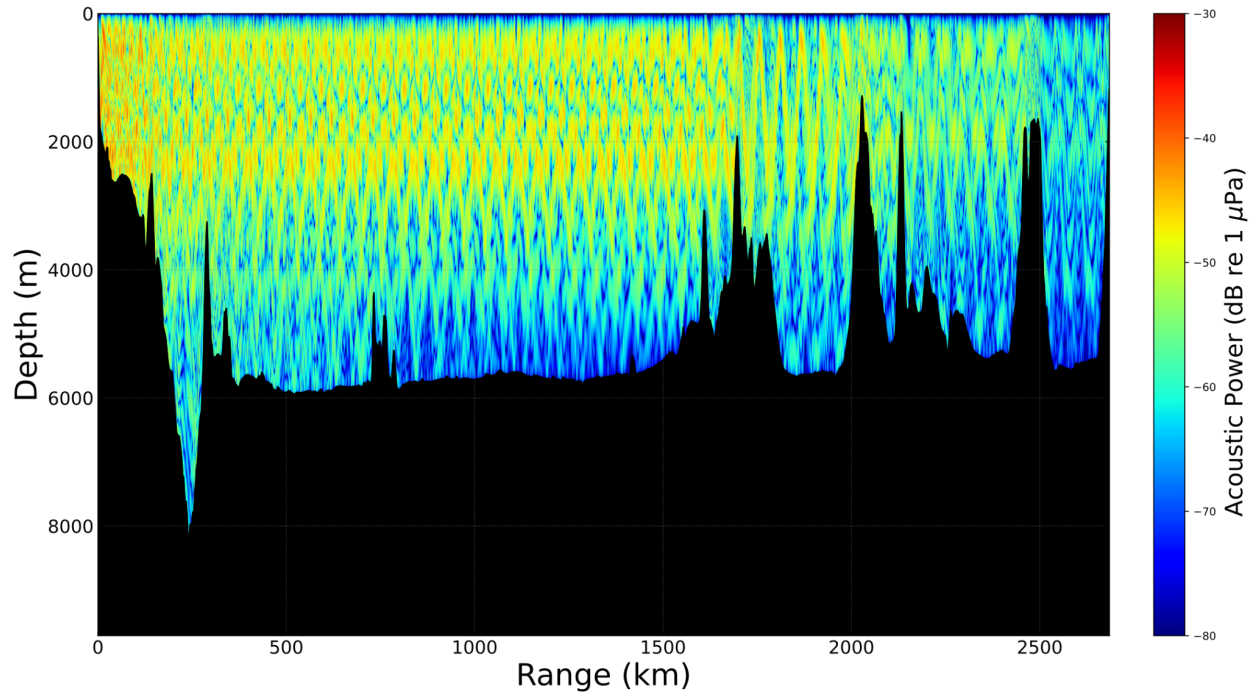


Supporting Figure S10. Infrasound InfraGA propagation Modeling from FOB to I39PW.

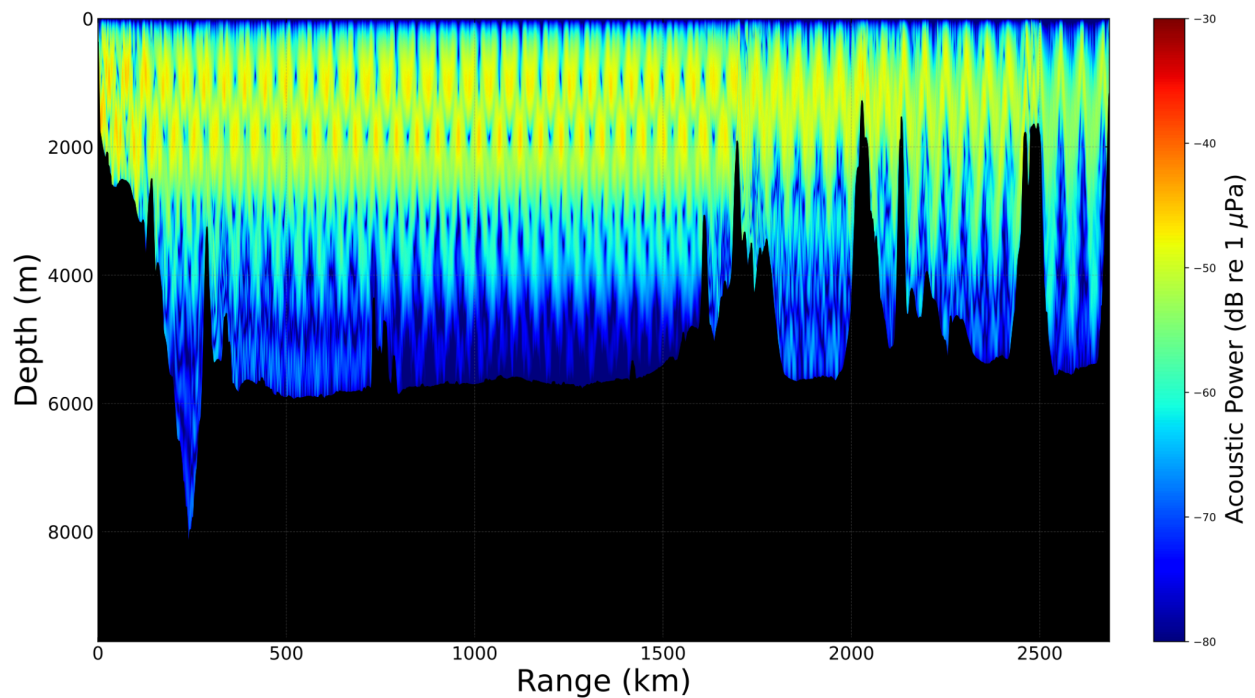
Upper Left) Sound speed with altitude in the direction of I39PW. Upper Right) Ray paths from FOB to I39PW for take-off angles from 20 to 50 degrees at 2-degree intervals. Lower Right) Trace velocity and celerity for rays at different ranges.



Supporting Figure S11. The 50 Hz hydroacoustic propagation modeling from FOB to H11S. Note that these results have no attenuation in the water column and include only attenuation from reflections at the seafloor. High-frequency sound waves will attenuate more than lower-frequency components.



Supporting Figure S12. The 10 Hz hydroacoustic propagation modeling from FOB to H11S. Note that these results have no attenuation in the water column and include only attenuation from reflections at the seafloor. High-frequency sound waves will attenuate more than lower-frequency components.



Supporting Figure S13. The 5 Hz hydroacoustic propagation modeling from FOB to H11S. Note that these results have no attenuation in the water column and include only attenuation from

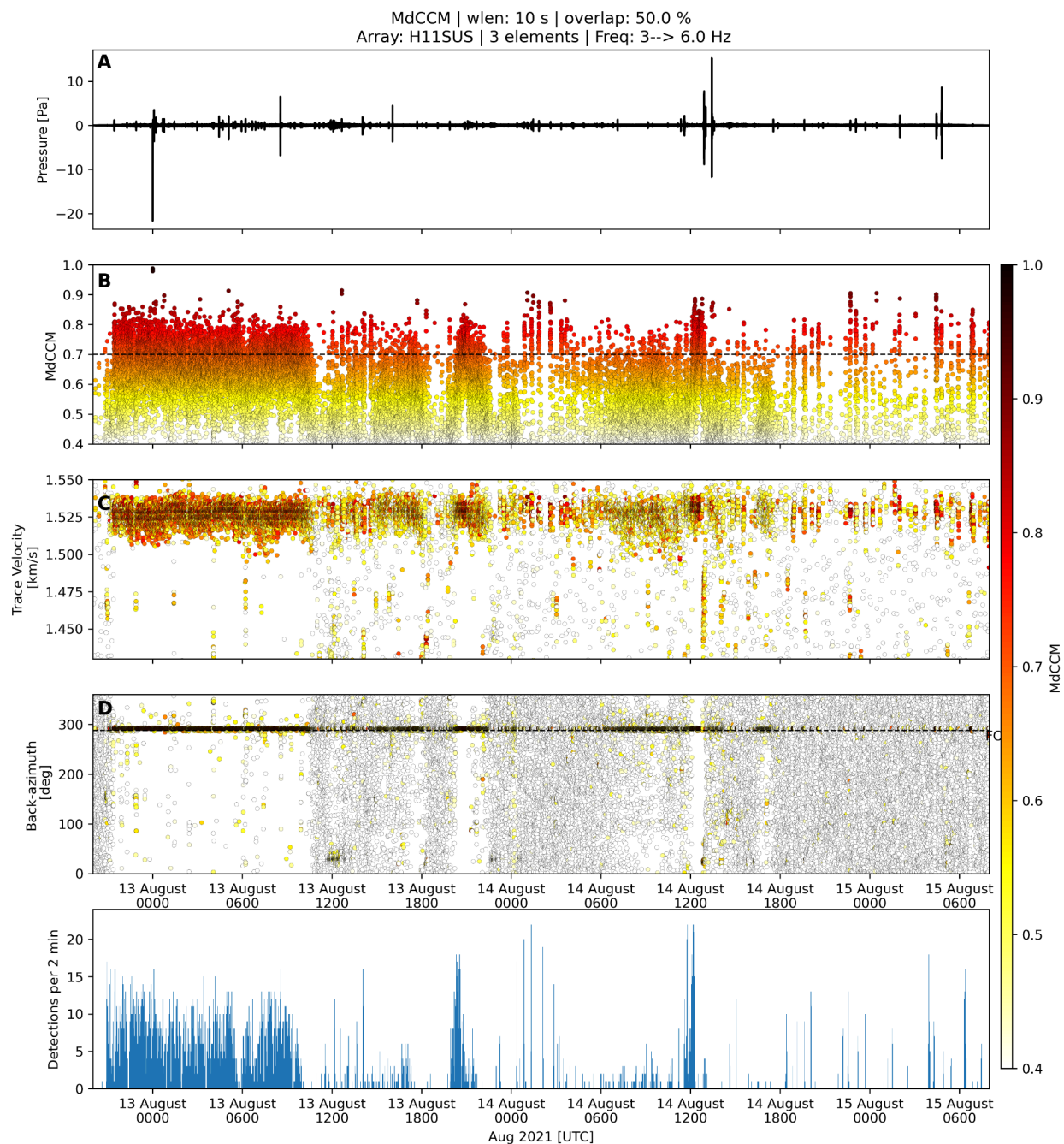
reflections at the seafloor. High-frequency sound waves will attenuate more than lower-frequency components.

Discussion on Density-driven vs. Passive transport

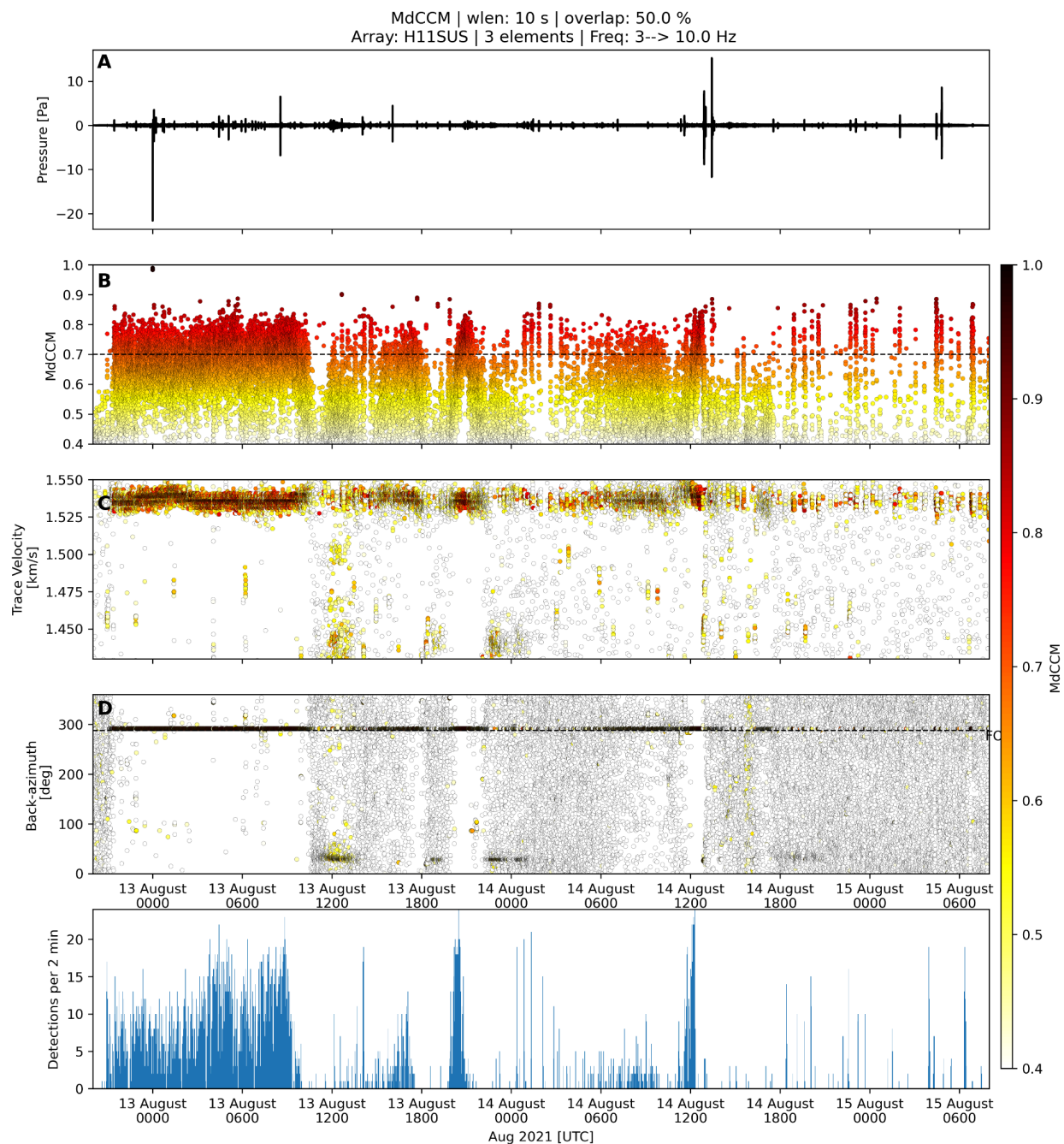
Volcanic plumes spread at an altitude of neutral buoyancy due to the density contrast between the plume material and the atmosphere, a gravity current. If the plume is in a strong wind field, the upwind plume will stall, and the downwind plume's speed will approach that of the wind. The spreading mechanism transitions from density-driven to passive transport from the vent to the downwind plume. Himawari captures the entirety of the plume, and we estimated the volume flux for each width and distance observation. However, the gravity current assumption is no longer valid at some distance; thus, the flux estimates are invalid. Costa et al. (2013) give the critical time scales at which the plume expansion regime transitions from density-driven to a combined density-driven and passive transport, t_b , and then from the combined regime to purely passive transport, t_p :

$$t_b = \frac{4\lambda Nq}{9\pi u_w^3}$$
$$t_p = \frac{32\lambda Nq}{9\pi u_w^3}$$

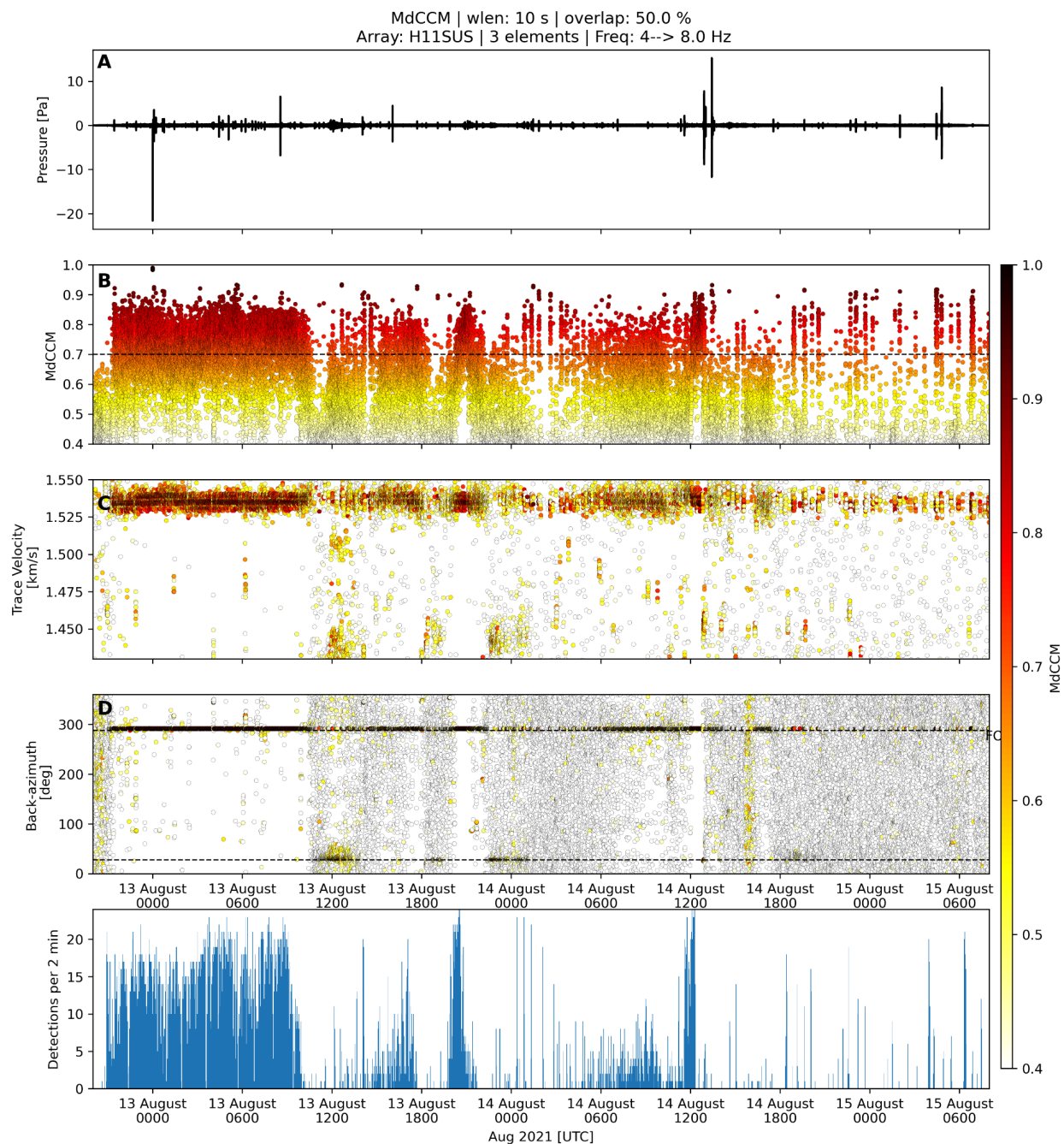
We then multiply these times by the windspeed to estimate the critical distances along the plume at which these transitions occur. FigVolFluxA shows our volume flux estimates with the critical distance d_b , solid black line, and d_p , dotted black line. These critical distance estimates show that most of our volume flux measurements are for a passively transported plume, which does not satisfy the gravity current assumption. During the peak of the eruption early on 13 August and later that same day at ~20:00 UTC, the critical distances highlight the transition from density-driven to a combined regime to passive transport. The low number of volume flux estimates that satisfy the gravity current assumption suggests that most of our volume fluxes are underestimated. There were many instances when Himawari observed the plume, but it was not attached to the vent. A higher satellite observation rate may allow us to capture more of the density-driven plume in the future.



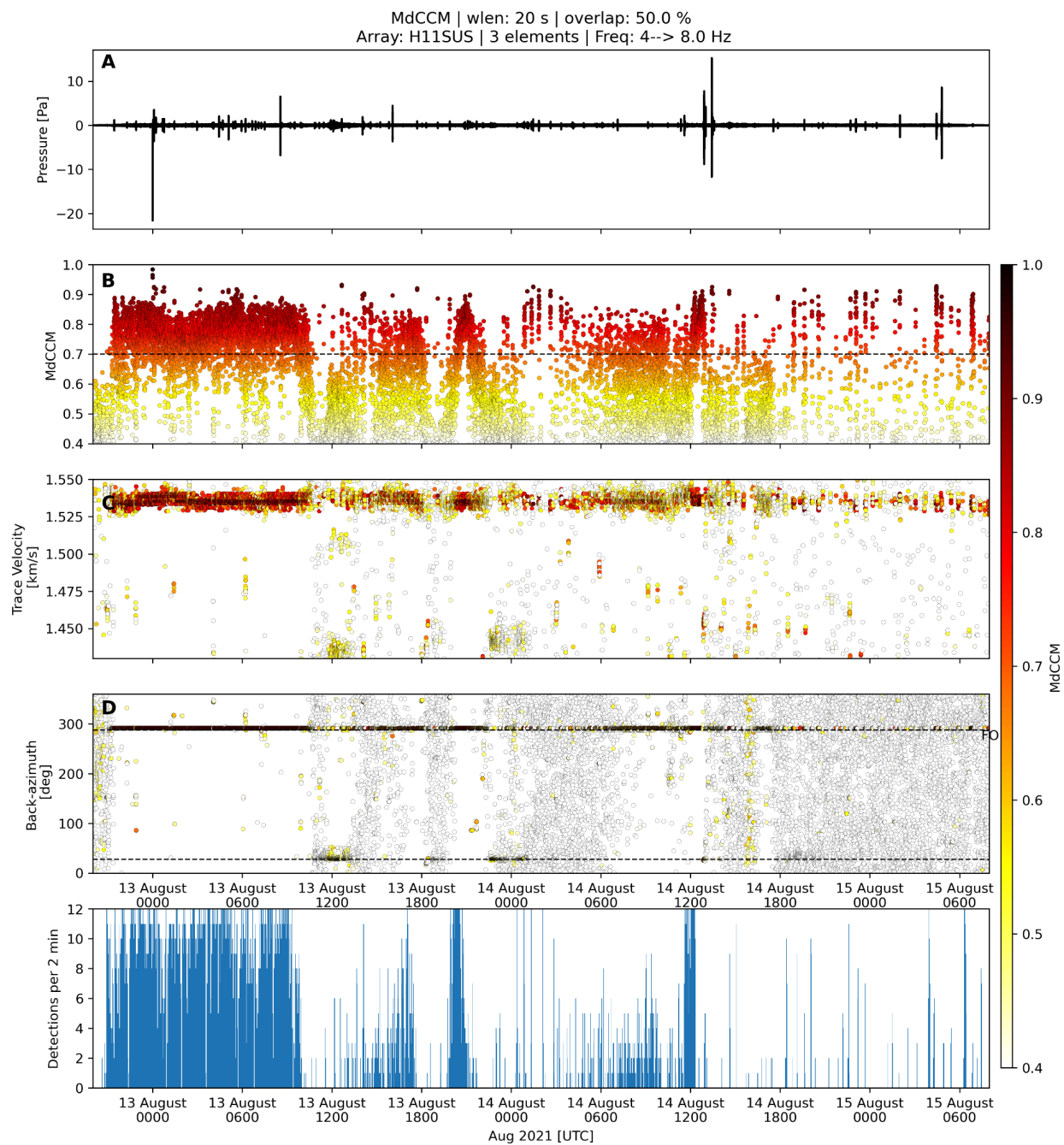
Supporting Figure S14. Array processing result for hydroacoustic array H11S using a 10s window, 50% overlap, frequency band 3-6 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.



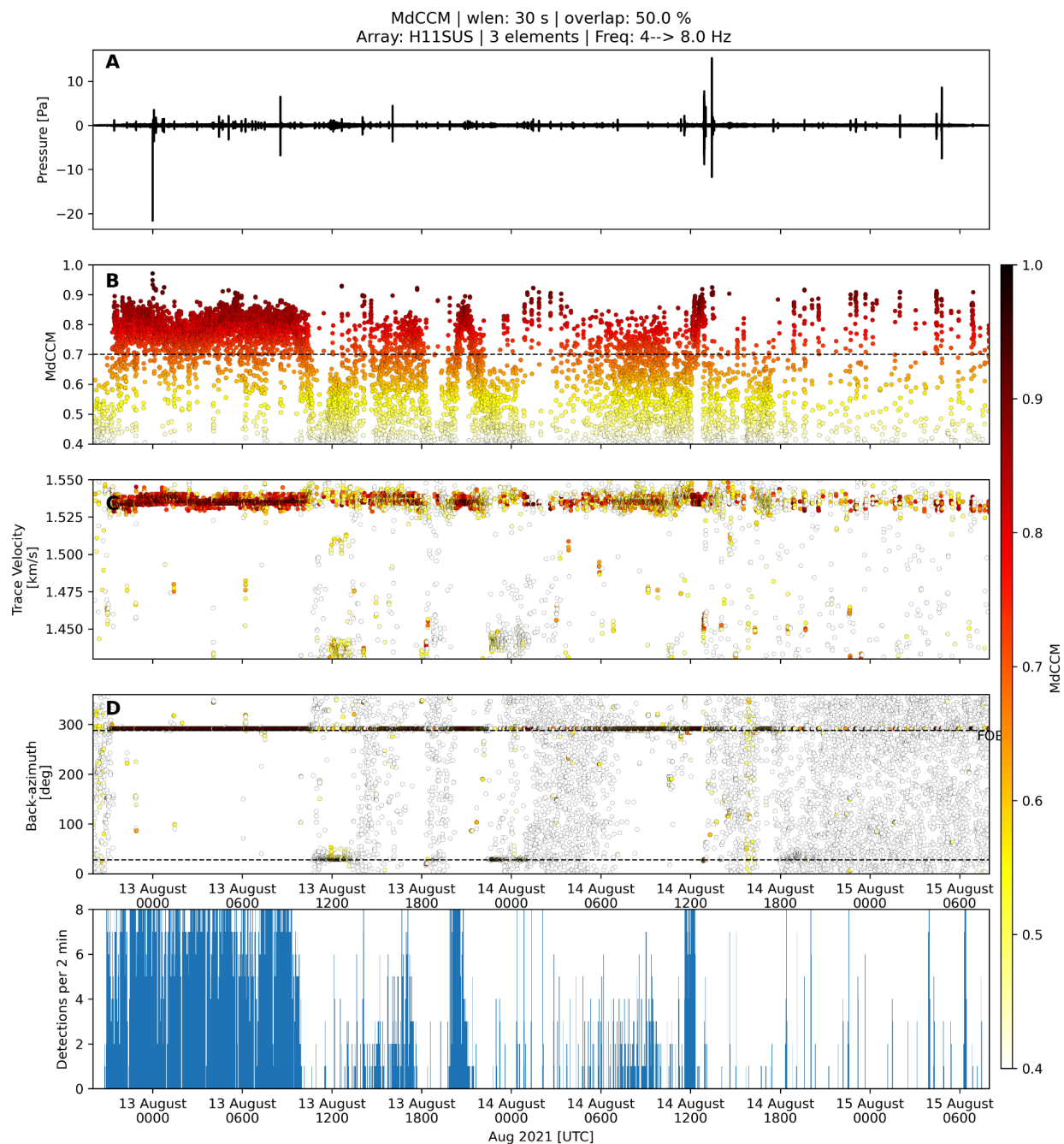
Supporting Figure S15. Array processing result for hydroacoustic array H11S using a 10s window, 50% overlap, frequency band 3-10 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.



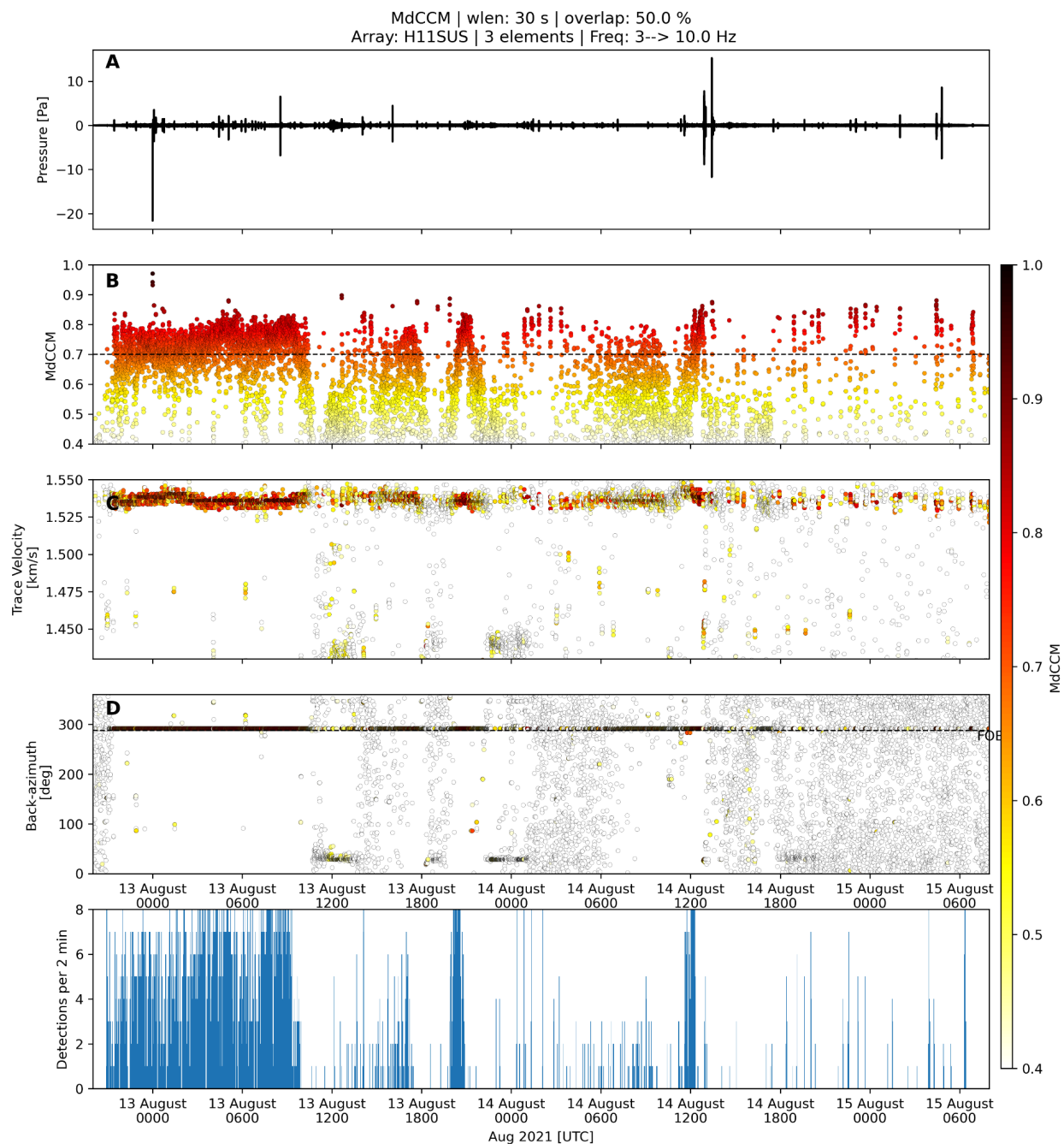
Supporting Figure S16. Array processing result for hydroacoustic array H11S using a 10s window, 50% overlap, frequency band 4-8 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.



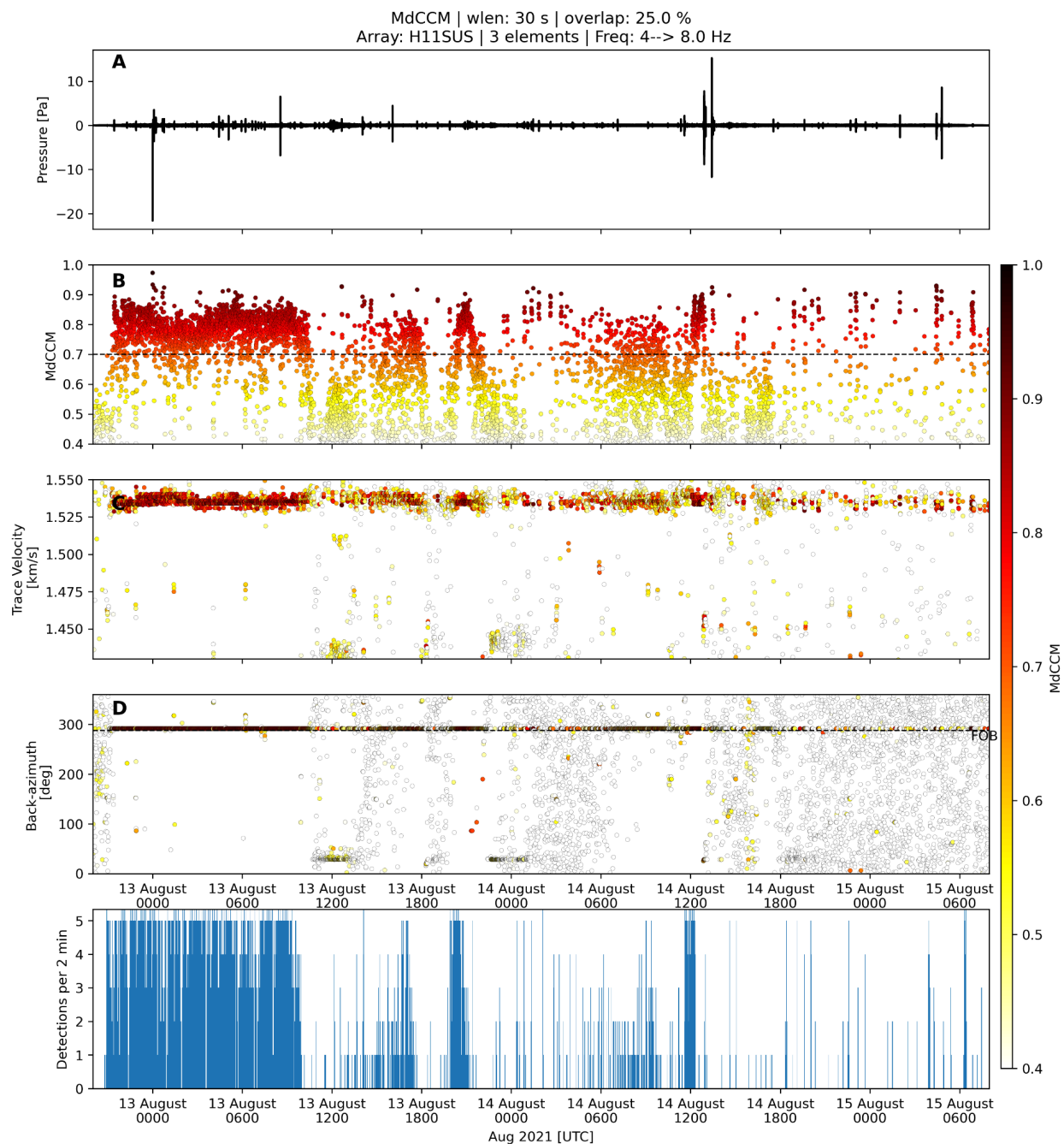
Supporting Figure S17. Array processing result for hydroacoustic array H11S using a 20s window, 50% overlap, frequency band 4-8 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.



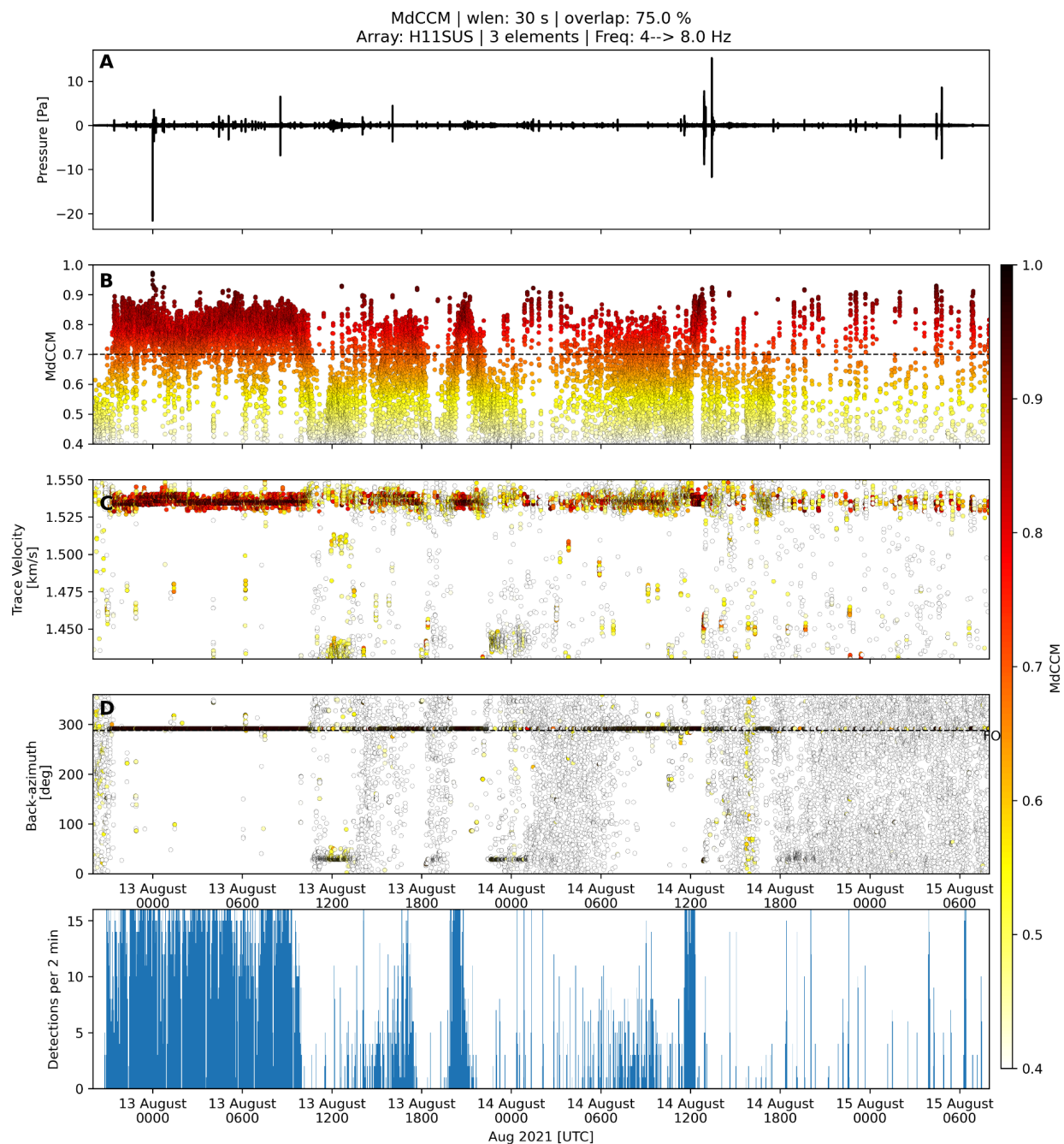
Supporting Figure S18. Array processing result for hydroacoustic array H11S using a 30s window, 50% overlap, frequency band 4-8 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.



Supporting Figure S19. Array processing result for hydroacoustic array H11S using a 30s window, 50% overlap, frequency band 3-10 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.



Supporting Figure S20. Array processing result for hydroacoustic array H11S using a 30s window, 25% overlap, frequency band 4-8 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.



Supporting Figure S21. Array processing result for hydroacoustic array H11S using a 30s window, 75% overlap, frequency band 4-8 Hz. A) Beamform from 3-element array data for FOB back-azimuth. B) Median cross-correlation maximum for each window. C) Trace velocity across the array. D) Back-azimuth resolved from cross-correlation lags. E) Histogram of detections per 2 minutes.